I. Model Problems.
II. Practice
III. Challenge Problems
VI. Answer Key

Web Resources

How to Solve Absolute Value Equations
www.mathwarehouse.com/absolute-value/how-to-solve-absolute-value-equation.php
I. Model Problems

The absolute value of a number is its distance from zero on the number line. For example the absolute value of 5, written |5|, is 5. Likewise, the absolute value of –5, written |–5| is also 5, because –5 is also 5 units away from zero on the number line. Absolute value is always positive; if the absolute value of a variable equals a negative number, the solution to the equation is “no solution.”

When solving absolute value equations, remember that there can be two solutions, because the absolute value of a number and its opposite are the same.

**Example 1** Solve |x| = 10.

\[ x = 10 \text{ or } x = -10 \]

**Definition of absolute value.**

**The answer is** \( x = 10 \text{ or } x = -10 \).

If the absolute value of an expression equals a number, solve by setting up two equations, one with the expression equal to the number and the other with the expression equal to the opposite of the number.

**Example 2** Solve |x + 2| = 7.

\[ x + 2 = 7 \text{ or } x + 2 = -7 \]

**Definition of absolute value.**

\[ x = 5 \text{ or } x = -9 \]

**Subtract.**

**The answer is** \( x = 5 \text{ or } x = -9 \).

Sometimes you need to isolate the absolute value expression before writing separate equations.
Example 3 Solve $3|x + 2| + 1 = 13$.

$3|x + 2| = 12$ Subtract.

$x + 2 = 4$ or $x + 2 = -4$

$x = 2$ or $x = -6$

Definition of absolute value. Subtract.

The answer is $x = 2$ or $x = -6$.

II. Practice

Solve. If there is no solution, write “no solution.”

1. $|x| = 8$
2. $|x + 6| = 9$
3. $|x - 3| = 8$
4. $|x + 9| = 12$
5. $|x - 1| = -4$
6. $|4x| = 24$
7. $\left|\frac{x}{3}\right| = 6$
8. $|2x + 1| = 25$
9. $2|x| = 80$
10. $|3x + 1| = 10$
11. $|x + 5| + 1 = 11$
12. $2|x| - 10 = 100$
13. $0.2|x| - 0.2 = 1.8$
14. $|x + 9| - 5 = -5$
15. $|x - 0.5| + 2 = 15$
16. $\left|x + \frac{2}{4}\right| = 7$
17. $|3x + 0.1| = 6$
18. $|3 - 2x| = 8$
19. $4|x - 2| = 8$
20. $|2x - 7| + 8 = 5$
21. \(0.5|x - 0.14| + 0.32 = 0.71\)  
22. \(-2|0.25x + 2| = 10\)  
23. \(-2|x| - 9 = -19\)  
24. \(4|2 - x| = 16\)  
25. \(\left|2x - \frac{1}{4}\right| = \frac{5}{8}\)  
26. \(\left|\frac{2}{3}x - 4\right| = 2\frac{1}{5}\)  
27. \(-3\left|x + \frac{1}{13}\right| = -\frac{12}{13}\)  
28. \(0.6|1 - 0.2x| = 0.15\)  
29. \(-3\left|x - \frac{6}{7}\right| = -30\)  
30. \(2\left|x + \frac{1}{9}\right| = \frac{2}{9}\)

III. Challenge Problems

31. What is the solution to the equation \(|x + 2| = -x|\)?

32. Does the equation \(|x + 2| = x\) have any solutions? Why or why not?

33. Correct the Error  
There is an error in the student work shown below:  
Question: Solve \(|x - 1| - 3 = 5\).  
Solution:  
\[x - 1 - 3 = 5 \text{ or } x - 1 - 3 = -5\]  
\[x - 4 = 5 \text{ or } x - 4 = -5\]  
\[x = 9 \text{ or } x = -1\]

What is the error? Explain how to solve the problem.
IV. Answer Key

1. 8 or -8
2. 3 or -15
3. 11 or -5
4. 3 or -21
5. no solution
6. 6 or -6
7. 18 or -18
8. 12 or -13
9. 40 or -40
10. 3 or -11/3
11. 5 or -15
12. 55 or -55
13. 10 or -10
14. -9
15. 13.5 or -12.5
16. 20 or -36
17. 1.97 or -2.03
18. 5.5 or -2.5
19. 4 or 0
20. no solution
21. 0.92 or -0.64
22. no solution
23. 5 or -5
24. -2 or 6
25. 7/16 or -3/16
26. 9.3 or 2.7
27. 3/13 or -5/13
28. 6.25 or -3.75
29. 112 or -28
30. 0 or -2/9
31. $x = -1$
32. No. The given equation can be separated into $x + 2 = x$ and $x + 2 = -x$. The first equation $x + 2 = x$ is equivalent to $2 = 0$, which has no solution. The second equation has $x = -1$ as a solution, but when -1 is plugged back into the original equation, it doesn’t work because the
absolute value cannot yield a negative number.
33. The student needed to isolate the absolute value before separating
the initial equation into two equations.